Finally the result will be :

$$P(\chi) = 1 - I(u, \chi) - [a_2 G_1 + a_3 G_2 + a_4 G_3 + \ldots]\psi(m, \chi).$$

This expression is also rigorous.

Remark.—The numerical valuation of $G_s\psi(m, x)$ is much simplified by the relation

$$G_s\psi(m, x) = (-1)^s \Delta^s \psi(m, x-s),$$

as there are tables which give the values of $\psi(m, z)$ *, and from them it is easy to compute the above differences.

II. Lexis's Problem.

4. Let us suppose that the probability of the single event is the same at each of n trials, and that it is equally likely that this probability is equal to p_1 or p_2 or $p_3 \ldots$ or p_ω ; let $q_1, q_2 \ldots q_\omega$ be the corresponding probabilities of failures.

If we denote by $P(x)$ the probability that the event will happen x times at n trials, we have

$$P(x) = \frac{1}{\omega} \sum_{i=1}^{\omega} \binom{n}{x} p_i^x q_i^{n-x}.$$

The generating function of this probability is

$$\phi(t) = \frac{1}{\omega} \sum_{i=1}^{\omega} (q_i + p_i t)^n. \quad . \quad . \quad . \quad . \quad (10)$$

The factorial momentum of $P(x)$ can easily be deduced from (10) by aid of formula (6)

$$\mathfrak{M}_s = \frac{1}{\omega} n(n-1)(n-2)\ldots(n-s+1) \sum_{i=1}^{\omega} p_i^s.$$

If we put $\frac{1}{\omega}\Sigma p_i = p$ and $np = m$, it results

$$\mathfrak{M}_0 = 1, \quad \mathfrak{M}_1 = m, \quad \mathfrak{M}_2 = \frac{1}{\omega} n(n-1)\Sigma p_i^2,$$

$$\mathfrak{M}_3 = \frac{1}{\omega} n(n-1)(n-2)\Sigma p_i^3, \text{ etc.}$$

The probability $P(x)$ can now be expanded in a series (4),

* K. Pearson, 'Tables for Statisticians and Biometricians,' pp. 113–121. Cambridge, 1914.

where the coefficients a_i are given by formula (5):

$$a_0 = 1, \quad a_1 = 0, \quad a_2 = \frac{1}{2}\left[\frac{1}{\omega} n(n-1)\Sigma p_i^2 - m^2\right],$$

$$a_3 = \frac{1}{6}\left[\frac{1}{\omega} n(n-1)(n-2)\Sigma p_i^3 - \frac{3}{\omega} mn(n-1)\Sigma p_i^2 + 2m^3\right],$$

etc.

Formula (9) gives, by aid of the above coefficients, the expansion of the probability that the number of successes in n trials does not exceed χ.

Both expansions are rigorous, and if the necessary number of terms are calculated, any prescribed precision can be obtained.

CXV. *The Difference in the Time Lags in the Disappearance of the Electric Double Refraction behind that of the Electric Field in several Liquids.* *By* J. W. Beams, *Ph.D., National Research Fellow, University of Virginia, and* Fred Allison, *Ph.D., Professor of Physics, Alabama Polytechnic Institute**.

When an electric field is applied across two metallic plates immersed in a liquid such as carbon bisulphide or chloroform, the liquid between the plates becomes doubly refracting. This phenomenon was discovered by Kerr † and is usually referred to as the "Kerr Effect." If the electric field is suddenly relaxed, the electric double refraction disappears very quickly. In the case of carbon bisulphide, Abraham and Lemoine ‡ have concluded that the electric double refraction follows without any lag the variations of the electric field which produces it. Their experiments, however, are subject to considerable uncertainty because of phenomena occurring at the beginning of the spark discharge which make their measurements unreliable §. Wood ‖, on the other hand, noted in his experiments that, when nitrobenzene was substituted for carbon bisulphide, the electric double refraction seemed to

* Communicated by the Authors.

† Kerr, Phil. Mag. vol. i. pp. 337, 446 (1875); vol. vii. pp. 85, 229 (1879).

‡ Abraham and Lemoine, *Comp. Rend.* t. cxxx. p. 499 (1900).

§ Brown and Beams, J. Opt. Soc. Amer. and Review of Scientific Instruments, vol. xi. p. 11 (1925); Beams, Phys. Rev. vol. xxviii. p. 475 (1926).

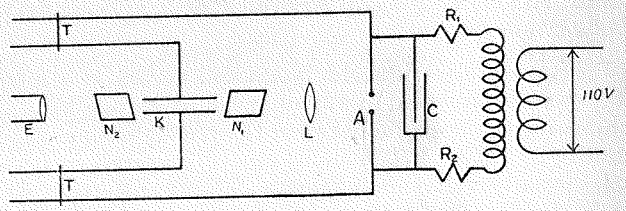
‖ Wood, Proc. Roy. Soc. A, vol. xcix. p. 371 (1921).

die away less rapidly. Recently *, preliminary mention of a method has been made by which the difference in the time lags of the electric double refraction behind the relaxation of the electric field could be detected; but the method did not lend itself to accurate quantitative measurements.

Since accurate quantitative data on these time lags are necessary for the application of the quick decay of electric double refraction to the measurement of short time intervals, as well as for the establishment of a satisfactory theory of the "Kerr Effect," the above-mentioned method has been improved and extended to the precise determination of the difference in the time lags in the disappearance of the electric double refraction behind that of the electric fields. This paper describes the method and gives the experimental results for carbon bisulphide, chloroform, and bromoform.

To understand the method of the experiment, consider fig. 1. K is a so-called "Kerr Cell" made by immersing

Fig. 1.



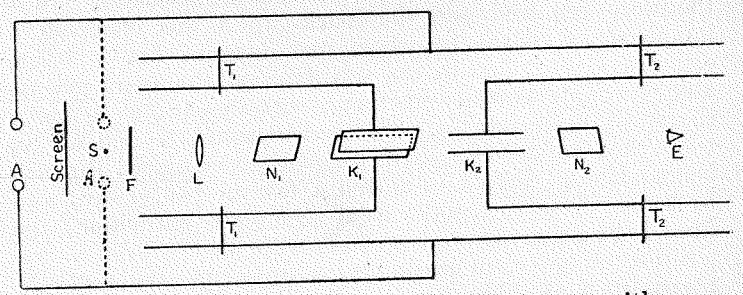
two parallel plates of metal in a liquid such as carbon bisulphide. N_1 and N_2 are crossed nicols, the diagonals making an angle of 45° with the plates of K. If an electric field is applied across the spark-gap A and consequently across the plates of K, light will pass N_2 so long as the field is applied, but no light will pass N_2 if the field is relaxed. It has been found † that if the electric field in K is relaxed by the spark at A, no light from A passes N_2 provided the lead wires are made very short. If, however, the lead wires from A to K are lengthened, light from A passes N_2, *i. e.*, the closing of K can be advanced or retarded with respect to a spark at A simply by shortening or lengthening the lead wires from A to K.

* Beams, J. Opt. Soc. Amer. and Review of Scientific Instruments, vol. xiii. p. 957 (1926).

† Brown and Beams, J. Opt. Soc. Amer. and R. S. I. vol. *xi.* p. 11 (1925); Beams, Phys. Rev. vol. xxviii. pl 475 (1926).

The arrangement of the apparatus used in this experiment is shown in fig. 2. K_1 and K_2 are "Kerr Cells" of practically identical dimensions, the plates being 12 cm. in length, 1 cm. in width, and 0·45 cm. apart. The plane of the plates of K_1 is vertical, while that of K_2 is horizontal. These cells are so constructed that the liquid with which they are filled can easily be removed and another substituted. N_1 and N_2 are crossed nicols as before, L is a lens which renders the light from a steady source S parallel, and F is a light filter which makes the light from S practically monochromatic. T_1 T_1 and T_2 T_2 are cross-wires by means of which the lead wires from the spark-gap A to K_1 and to K_2, respectively, can be lengthened or shortened by the observer at E. The lead wires from A to K_2 are so arranged that K_2 can be moved in the direction SN_1N_2 four or five metres without changing their length or their distances apart. The source of high

Fig. 2.



potential applied across the spark-gap A was either a 60-cycle, 10,000-volt transformer or an induction coil.

Suppose that K_1 and K_2 are first filled with carbon bisulphide and the high potential is slowly applied across A. The electric fields across K_1 and K_2 are therefore equal during the time of charging. Hence, since K_1 and K_2 are identical and the planes of their plates are at right angles, the electric double refraction in K_1 is exactly compensated by that in K_2 and no light from S passes N_2 during the charging. In the case of a sudden discharge of K_1 and K_2 through the spark-gap A, no light will pass N_2 if the lengths of the lead wires from A to K_1 are equal to, and symmetrical with, those from A to K_2, *i.e.*, the electric double refraction in K_1 at every instant is compensated by that in K_2. On the other hand, if K_2 by lengthening the lead wires from A to K_2 is made to discharge a certain time after K_1, light will pass N_2. If now K_2 is moved backward

in the direction AN_1N_2, (although not so shown in fig. 2, N_2 was about five metres behind N_1; the distance, of course, separating N_2 and N_1 is immaterial), it is possible to find a position of K_2 for which no light passes N_2. The distance through which K_2 must be moved backward to secure compensation was found to be equal to the total increase of lead wire from A to K_2, *i. e.*, the electric impulse travelled along the lead wires at approximately the velocity of light, a finding which is in agreement with many well-known results.

To increase the precision of these experiments a very strong source of light is required at S. It has been previously shown* that with the above arrangement of apparatus it is possible to pick out of the zinc spark the zinc-spark lines 4912·24 A. practically alone, and that they are very intense for a few hundred millionths of a second, depending upon the auxiliary capacity in parallel with the spark. The steady light-source S was therefore removed and the spark-gap A was clamped in its place, as shown by the dotted lines in fig. 2. Zinc electrodes were put in the terminals of the spark-gap A, and a variable capacity was connected in parallel to make the spark more intense. The lead wires AT_1K_1 and AT_2K_2 were then adjusted so that, when K_2 was placed directly behind K_1 and T_2T_2 were moved a few metres forward and backward from the position of complete extinction, the lines 4912·24 A. passed N_2. T_2T_2 were then moved to the position of complete extinction, *i. e.*, where the nicol N_2 transmitted no light. Chloroform was then substituted in K_2 for carbon bisulphide, and the plane of K_2 was rotated through an angle of 90°. (This adjustment is necessary, since chloroform behaves in an electric field as a negative uniaxial crystal and carbon bisulphide as a positive uniaxial crystal.) It was then found that light passed N_2, but that if K_2 was moved backward a distance of 100 cm., practically no light passed N_2, *i. e.*, the electric double refraction in carbon bisulphide is practically compensated by that in chloroform, resulting from the fact that the Kerr constant of carbon bisulphide is almost equal to that of chloroform. (It is not necessary that the Kerr constants of two liquids be equal in order to secure compensation when monochromatic light is used, since by rotating the planes of the plates of K_1 and of K_2 with respect to N_1 the magnitude of the double refraction in K_1 and K_2 can be varied.)

When the two cells contained carbon bisulphide and the

* Beams, *loc. cit.*

lead wires were so adjusted that N_2 extinguished the light, K_2 being immediately behind K_1, the electric fields in K_1 and K_2 were then relaxed together; or, to be exact, the electric field in K_1 was relaxed before that in K_2 by a time equal to the distance between the centres of the cells divided by the velocity of light. When chloroform was substituted for carbon bisulphide, the electric fields in K_1 and K_2 were still removed practically together as in the case of carbon bisulphide; but since it was found necessary to move K_2 back a distance of 100 cm. in order to extinguish the light through N_2, the disappearance of the electric double refraction in chloroform must lag behind that in carbon bisulphide a time equal to 100 cm. divided by the velocity of light, or $3{\cdot}3 \times 10^{-9}$ sec. Similar experiments on the determination of time-lag differences of the electric double refraction behind the relaxing electric field were made for Bromoform. Table I. gives the results, together with the Kerr constants, the dielectric constants, and the coefficients of viscosity.

TABLE I.

Liquid.	Dielectric Constant.	Kerr Constant.	Coefficient of Viscosity at 20° C.	Difference in time lag of Kerr effect behind electric field.
Carbon bisulphide, CS_2.	2·6	4·28 ,,	0·00367	
				100 cm. $3{\cdot}3\times10^{-9}$ sec.
Chloroform, $CHCl_3$...	5·2	−4·01 ,,	0·00564	
				0 cm. 0 ,,
Bromoform, $CHBr_3$..	4·4	−3·8 ,,	0·20	

It will be noted from the table that the time lag in the liquids investigated does not depend, directly at any rate, upon the Kerr constant, the dielectric constant of the liquid, or the coefficient of viscosity. A great number of liquids, however, must be tested before definite conclusions can be drawn.

The fact that it was almost possible to extinguish completely the light through N_2 indicates that the double refraction remains constant for a definite time after the electric field is removed, and then decays abruptly. These abrupt decays must be similar in the cases so far investigated; otherwise a rather bright minimum would have occurred instead of almost complete extinction. The lack of complete extinction, however, probably indicates a small difference in the decay curves, and we hope to investigate this point further. The occurrence of this faint minimum instead of complete extinction might be

the result of the changes in capacity introduced in K_2 when the liquids were changed. However, this effect is very small and probably cannot completely account for the observed lack of perfect compensation.

We wish to thank Professor L. G. Hoxton for his interest in the work and for placing apparatus at our disposal.

Summary.

1. A method has been developed by means of which the difference in time lags of the disappearance of the electric double refraction after the removal of the electric field in various liquids can be measured with a precision of $0{\cdot}4 \times 10^{-9}$ sec.

(2) Of the liquids investigated, carbon bisulphide had the smallest lag, and is therefore probably the best liquid to use in experiments in which the decay of electric double refraction is used to measure small time intervals.

Rouse Physical Laboratory,
University of Virginia,
August 31, 1926.

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SEVENTH SERIES.]

APRIL 1927.

LXXXIV. *Electrostatic Problems concerning certain Inverted Spheroids.*

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Introduction.

1. IN this paper we consider an analytical treatment of various problems of electrostatics associated with the surfaces obtained by inverting an oblate spheroid with respect to a point on the axis of symmetry.

We may take coordinates (ξ, η, ω) defined by the relations:

$$z' = a'(\cos\eta \sinh\xi + \sinh\xi_1),$$
$$x' = a' \sin\eta \cosh\xi \cos\omega,$$
$$y' = a' \sin\eta \cosh\xi \sin\omega.$$

Then the surfaces ξ = constant are the confocal oblate spheroids:

$$(z'-a'\sinh\xi_1)^2/a'^2\sinh^2\xi + (x'^2+y'^2)/a'^2\cosh^2\xi = 1,$$

and the surfaces η=constant are the confocal hyperboloids of revolution of one sheet:

$$(z'-a'\sinh\xi_1)^2/a'^2\cos^2\eta - (x'^2+y'^2)/a'^2\sin^2\eta = 1.$$

The centre of coordinates $(x'\,y'\,z')$ is then at a distance $(-a'\sinh\xi_1)$ from the centre of the spheroids in the direction of the minor axes. The particular spheroid $\xi=\xi_0$ will

* Communicated by the Author.